Coordinate Transformations

Course *Reference Systems for Geoinformation*

Lab 3

Goals
To familiarize yourself with elementary methods for computing coordinate transformations, and to learn to apply similarity and affine transformations in a practical GIS context.

1st Part (8 Points)
1. Given the following WGS84 geographic coordinates, compute geodetic coordinates in the (West) German Bessel ellipsoid-based system using the (direct) *Abridged Molodensky* formulae and an indirect *three-parameter geocentric transformation*:
   Latitude   51° 56' 28.1"
   Longitude   7° 36' 37.1"
   Height   66.0 m
   Use the following shifts between the ellipsoid centers: ΔX= -583m, ΔY= -68m, ΔZ= -395m. (6 Points)
2. Compare your results with those of the online transformation service at
   https://upd.geodatenzentrum.de/geodaten/gdz_rahmen.gdz_div?gdz_spr=eng&gdz_akt_zeile=3&gdz_anz_zeile=6&gdz_user_id=0
   (2 Points)

2nd Part (12 Points)
To show the road network of Münster on top of a raster image for a routing application, you could transform the road data (e.g. from ATKIS) to the raster image.
1. Describe the reference system of the raster image and which problems this may pose. (1 pt)
2. Transform the road data into the reference system of the raster image using a similarity transformation. (4 pts)
3. Transform the road data into the reference system of the raster image using an affine transformation. (4 pts)
4. Compare the two transformation results by measuring some coordinate values. How do you explain the magnitude and direction of the coordinate differences? (3 pts)

Use the minimal set of necessary common points between the two data sets and measure their coordinate values in any GIS. Submit all data, calculations and results in a technical report.

Schedule
Handout: May 10th
Lab sessions: May 10th and 17th
Submission: **May 27th** until 12:00 o'clock (noon). One more week to work on the lab, but do start early, and there are only two sessions to ask questions!

Reference Systems for Geographic Information

"Coordinate Transformations"

Technical Report on Lab 3

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Date of Submission

25th May, 2010

1st Part

1. (West) German Bessel ellipsoid-based system using the (direct) Abridged Molodensky formulae

Given,

WGS84 geographic coordinates:

Latitude, $\varphi_{WGS84} = 51° 56' 28.1'' = 51.941139°$

Longitude, $\lambda_{WGS84} = 7° 36' 37.1'' = 7.610306°$

Height, $h_{WGS84} = 66.0$ m

$\Delta X = -583$ m, $\Delta Y = -68$ m and $\Delta Z = -395$ m

We know,

For Earth Ellipsoid WGS84:

$a_{WGS84} = 6378137$ m, $1/f = 298.257223563$, $f_{WGS84} = 0.0033528106647$

For Bessel Ellipsoid 1841:

$a_{Bessel} = 6377397.155$ m, $1/f = 299.1528153513233$, $f_{Bessel} = 0.0033427731537$, $b_{Bessel} = 6356078.963$ m

$\Delta a = a_{Bessel} - a_{WGS84} = 6377397.155$ m $- 6378137$ m $= -739.845$

$\Delta f = f_{Bessel} - f_{WGS84} = 0.0033427731537 - 0.0033528106647 = -0.000010037511$

Again, we know:

Eccentricity e is defined as,

$e^2 = (2*f_{WGS84}) - (f_{WGS84})^2 = \mathbf{2 \cdot (0.0033528106647) - (0.0033528106647)^2}$

$= 0.00669437999$

The radius of curvature in the meridional section,

$\rho = [a_{WGS84} * (1-e^2)] / (1-e^2\sin^2 \varphi)^{3/2}$

$= [6378137 * (1-0.00669437999)] / [1-((0.00669437999)^2 * \sin (51.941139^2)]^{3/2}$

$= 6335439.32729372 / [1- (0.0000448147234505124001 * 0.6199638906726)]^{3/2}$

$= 6335439.32729372 / (0.9999722164897)^{3/2}$

$= 6335439.32729372 / 0.9999907387441$

$= 6335498.00196197$

The radius of curvature in the prime vertical,

$$\nu = a_{WGS84} / (1-e^2\sin^2\varphi)^{1/2}$$

$$= 6378137 / [1-((0.00669437999)^2 * \sin(51.941139)^2)]^{1/2}$$

$$= 6378137 / (0.9999722164897)^{1/2}$$

$$= 6378137 / 0.9999861081484$$

$$= 6378225.60536358$$

$$\Delta\varphi'' = (-\Delta X\sin\varphi\cos\lambda - \Delta Y\sin\varphi\sin\lambda + \Delta Z\cos\varphi + (a\Delta f + f\Delta a)\sin2\varphi) / \rho\sin1''$$

Here, $\sin1'' = \sin(1/3600) = $ **4.8481368110764** $\cdot$ **10^{-6}**

$\rho\sin1'' = 6335498.00196197 * $ **4.8481368110764** $\cdot$ **10^{-6}**

$$= 30.7153610798128$$

$-\Delta X\sin\varphi\cos\lambda = -(-583) * \sin(51.941139) * \cos(7.610306) = 454.997933554926$

$-\Delta Y\sin\varphi\sin\lambda = -(-68) * \sin(51.941139) * \sin(7.610306) = 7.0907772423958$

$\Delta Z\cos\varphi = -395 * \cos(51.941139) = -243.505921812617$

$(a\Delta f + f\Delta a)\sin2\varphi$

$$= [(6378137*(-0.000010037511)) + (0.0033528106647*(-739.845))]*\sin2(51.941139)$$

$$= (-66.501180503232) * 0.9707907394175$$

$$= -64.5587301928692$$

$(-\Delta X\sin\varphi\cos\lambda - \Delta Y\sin\varphi\sin\lambda + \Delta Z\cos\varphi + (a\Delta f + f\Delta a)\sin2\varphi$

$$= 454.997933554926 + 7.0907772423958 - 243.505921812617 - 64.5587301928692$$

$$= 154.024058791836$$

$$\Delta\varphi'' = (-\Delta X\sin\varphi\cos\lambda - \Delta Y\sin\varphi\sin\lambda + \Delta Z\cos\varphi + (a\Delta f + f\Delta a)\sin2\varphi) / \rho\sin1''$$

$$= 154.024058791836/30.7153610798128$$

$$= 5.014561228553 \text{ arc seconds}$$

$$= 0.0013929°$$

$\mathbf{\phi_{Bessel}} = \phi_{WGS84} + \Delta\ \phi"$

$= 51.941139° + 0.0013929°$

$= 51.9425319°$

$= \mathbf{51°\ 56'\ 33.1146"}$

$\Delta\ \lambda" = (-\Delta X \sin\ \lambda + \Delta Y \cos\ \lambda)\ /\ v\cos\ \phi\ \sin 1"$

$-\Delta X \sin\ \lambda = -(-583) * \sin\ (7.610306) = 77.2094194058205$

$\Delta Y \cos\ \lambda = -68 * \cos\ (7.610306) = -67.4010379649823$

$v\cos\ \phi\ \sin 1" = 6378225.60536358 * \cos\ (51.941139) *\ \mathbf{4.8481368110764 \cdot 10^{-6}}$

$= 19.0628212322445$

$\Delta\ \lambda" = (77.2094194058205 - 67.4010379649823)\ /\ 19.0628212322445$

$= 0.5145293721922$ arc seconds

$= 0.000142925°$

$\mathbf{\lambda_{Bessel}} = \lambda_{WGS84} + \Delta\ \lambda" = 7.610306° + 0.000142925° = 7.610448925° = \mathbf{7°\ 36'\ 37.6164"}$

$\Delta\ h = \Delta X \cos\phi\cos\ \lambda + \Delta Y \cos\phi\sin\ \lambda + \Delta Z \sin\phi + (a\Delta f + f\Delta a)\ \sin^2\phi - \Delta a$

$\Delta X \cos\phi\cos\ \lambda = -583 * \cos\ (51.941139) * \cos\ (7.610306) = -356.236699422959$

$\Delta Y \cos\phi\sin\ \lambda = -68 * \cos\ (51.941139) * \sin\ (7.610306) = -5.5516627546828$

$\Delta Z \sin\phi = -395 * \sin\ (51.941139) = -311.014253760479$

$(a\Delta f + f\Delta a)\ \sin^2\phi$

$= [(6378137 * (-0.000010037511)) + (0.0033528106647 * (-739.845))] * \sin\ (51.941139)^2$

$= (-66.501180503232) * 0.6199638906726$

$= -41.2283305991046$

$\Delta\ h = -356.236699422959 - 5.5516627546828 - 311.014253760479 - 41.2283305991046 + 739.845 = 25.8140534627746$ m

$\mathbf{h_{Bessel}} = h_{WGS84} + \Delta\ h = 66.0\ m + 25.8140534627746\ m = \mathbf{91.8140534627746\ m}$

Indirect three-parameter geocentric transformation

$X_{WGS84} = (v + h) \cos\varphi \cos\lambda$

$= (6378225.60536358 + 66) * \cos(51.941139) * \cos(7.610306)$

$= 3897395.4536053$ m

$Y_{WGS84} = (v + h) \cos\varphi \sin\lambda$

$= (6378225.60536358 + 66) * \cos(51.941139) * \sin(7.610306)$

$= 520737.116823571$ m

$Z_{WGS84} = [((1 - e^2) * v) + h] \sin\varphi$

$= ((1 - 0.00669437999) * 6378225.60536358) + 66) * \sin(51.941139)$

$= [(0.99330562001 * 6378225.60536358) + 66] * \sin(51.941139)$

$= (6335527.33949933 + 66) * \sin(51.941139)$

$= 4988505.91041582$ m

$X_{Bessel} = X_{WGS84} + \Delta X = 3897395.4536053 - 583 = 3896812.4536053$ m

$Y_{Bessel} = Y_{WGS84} + \Delta Y = 520737.116823571 - 68 = 520669.116823571$ m

$Z_{Bessel} = Z_{WGS84} + \Delta Z = 4988505.91041582 - 395 = 4988110.91041582$ m

Bessel Geocentric X, Y, Z to Geographic Bessel Coordinate Reference System

$p = (X^2_{Bessel} + Y^2_{Bessel})^{1/2}$

$= [(3896812.4536053)^2 + (520669.116823571)^2]^{1/2}$

$= \sqrt{15456243627787}$

$= 3931442.94474522$

$e_{Bessel}{}^2 = 2 * f_{Bessel} - f^2_{Bessel} = (2 * 0.0033427731537) - (0.0033427731537)^2$

$= 0.006674372175$

$\varepsilon = e_{Bessel}{}^2 / (1-e_{Bessel}{}^2) = (0.006674372175)^2 / [1- (0.006674372175)^2]$

$= \mathbf{4.4549228475761 \cdot 10^{-5}}$

$\tan(u) = (Z_{Bessel} * a_{Bessel}) / (p * b_{Bessel})$

$= (4988110.91041582 * 6377397.155) / (3931442.94474522 * 6356078.963)$

$= 31811164328910.3103349921 / 24988561795329.86423680686$

$= 1.2730290198156$

$u = \tan^{-1}(1.2730290198156) = 51.8493716535407$

$\tan\varphi_{Bessel} = (Z_{Bessel} + \varepsilon * b_{Bessel} * \sin^3 u) / (p - e_{Bessel}{}^2 * a_{Bessel} * \cos^3 u)$

$= [4988110.91041582 + \{\varepsilon * 6356078.963 * \sin(51.8493716535407)^3\}] /$

$[3931442.94474522 - \{0.006674372175 * 6377397.155 * \cos(51.8493716535407)^3\}]$

$= [4988110.91041582 + 137.702733192869] / [3931442.94474522 - 10033.4909079355]$

$= \dfrac{4988248.61314901}{3921409.45383728}$

$= 1.2720550281399$

$\varphi_{Bessel} = \tan^{-1}(1.2720550281399) = 51.8280666223331° = \mathbf{51° \; 49' \; 41.0406''}$

$\tan\lambda_{Bessel} = Y_{Bessel}/X_{Bessel} = 520669.116823571 / 3896812.4536053$

$= 0.1336141071767$

$\lambda_{Bessel} = \tan^{-1}(0.1336141071767) = 7.6104489446809° = \mathbf{7° \; 36' \; 37.6164''}$

$v_{Bessel} = a_{Bessel} / (1 - e^{2*}\sin^2\varphi)^{1/2}$

$= 6377397.155 / [1 - \{0.006674372175 * \sin(51.8280666223331)^2\}]^{1/2}$

$= 6377397.155 / (1-0.0041250765004)^{1/2}$

$= 6377397.155 / 0.9979353303194$

$= 6390591.61575014 \ m$

$h_{Bessel} = p \sec \varphi - v = (p / \cos \varphi) - v$

$= \{3931442.94474522 / \cos (51.8280666223331)\} - 6390591.61575014$

$= 6361317.65505726 - 6390591.61575014$

$\mathbf{h_{Bessel} = -29273.96069288 \ m}$

2. Coordinate Transformation Service (CTS)

Figure: Online Coordinate Transformation Service (CTS)

Table: Comparison of different methods used to get the coordinates

	Latitude (φ)	Longitude (λ)	Height (h)
WGS84 Geographic (Given)	51° 56' 28.1"	7° 36' 37.1"	66.0 m
Bessel ellipsoid (Abridged Molodensky)	51° 56' 33.1146"	7° 36' 37.6164"	91.814 m
Bessel ellipsoid (indirect three-parameter geocentric)	51° 49' 41.0406"	7° 36' 37.6164"	-29273.961 m
Bessel ellipsoid (through the given online transformation service)	51° 56' 33.0432"	7° 36' 40.14"	--------

Observations:

i. The heights are significantly different, may be due to different datums.
ii. Longitude values are almost same.
iii. Latitude values are slightly changing.

2nd Part

1. Describe the reference system of the raster image and which problems this may Pose?

Answer: The raster image has no reference system. It has cell grids divided into columns/ width and row/ height. Therefore, if we open this raster image in any software, it is not possible to locate any place or point by coordinates.

To solve this problem we have to georeference this raster image.

2. General Equations for Similarity Transformation:

$$Xn = a \cdot X_0 + b \cdot Y_0 + c$$
$$Yn = -b \cdot X_0 + a \cdot Y_0 + d$$

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} X_0 & Y_0 & 1 & 0 \\ Y_0 & -X_0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

With four parameters to be determined, there is a requirement for at least two points of known 2D coordinates (Source: Iliffe & Lott, 2000, p-115).

For solving part two of this assignment, we have determined/measured the coordinates of 5 different points (Control Points) both from the shapefile and the raster image. The positions of the points are shown at the end.

Here, the Image Resolution = (Width, Height) = (3529, 3438)

Control Point	X_o (Shape)	Y_o (Shape)	X_n (Image)	Y_n (Image)
1	3405650.201	5758878.930	2668	1461 (3438-1461=**1977**)
2	3405415.509	5758618.090	2597	1541 (3438-1541=**1897**)
3	3405960.296	5757978.003	2769	1721 (3438-1721=**1717**)

For this similarity transformation we have considered Control Points 1 and 2. Control Point (CP) 3 has been used for testing the parameter values through this transformation.

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} * \begin{bmatrix} Xo \\ Yo \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

$$2668 = a \cdot 3405650.201 + b \cdot 5758878.930 + c$$
$$1977 = -b \cdot 3405650.201 + a \cdot 5758878.930 + d$$
$$2597 = a \cdot 3405415.509 + b \cdot 5758618.090 + c$$
$$1897 = -b \cdot 3405415.509 + a \cdot 5758618.090 + d$$

$$\begin{bmatrix} 2668 \\ 1977 \\ 2597 \\ 1897 \end{bmatrix} = \begin{bmatrix} 3405650.201 & 5758878.930 & 1 & 0 \\ 5758878.930 & -3405650.201 & 0 & 1 \\ 3405415.509 & 5758618.090 & 1 & 0 \\ 5758618.090 & -3405415.509 & 0 & 1 \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 3405650.201 & 5758878.930 & 1 & 0 \\ 5758878.930 & -3405650.201 & 0 & 1 \\ 3405415.509 & 5758618.090 & 1 & 0 \\ 5758618.090 & -3405415.509 & 0 & 1 \end{bmatrix}^{-1} * \begin{bmatrix} 2668 \\ 1977 \\ 2597 \\ 1897 \end{bmatrix} = \begin{bmatrix} 0.3048326 \\ -0.002077034 \\ -1023524 \\ -1760591 \end{bmatrix}$$

R-Codes:

> A <-matrix (c (3405650.201, 5758878.930, 3405415.509, 5758618.090, 5758878.930, -3405650.201, 5758618.090, -3405415.509, 1,0,1,0, 0, 1, 0, 1), 4)

> X<-c (2668, 1977, 2597, 1897)

```
> A
        [,1]       [,2] [,3] [,4]
[1,] 3405650   5758879    1    0
[2,] 5758879  -3405650    0    1
[3,] 3405416   5758618    1    0
[4,] 5758618  -3405416    0    1
> X
[1] 2668 1977 2597 1897
> B=A^-1*X
> B<-solve(A,X)
> B
[1]   3.048326e-01 -2.077034e-03 -1.023524e+06 -1.760591e+06

> p_a<-B[1]
> p_a
[1] 0.3048326
>
> p_b<-B[2]
> p_b
[1] -0.002077034
>
> p_c<-B[3]
> p_c
[1] -1023524
>
>
> p_d<-B[4]
> p_d
[1] -1760591

> µ <- sqrt(0.3048326^2 + -0.002077034^2)
> µ
[1] 0.3048255
>
> α <- atan(-0.002077034 / 0.3048326) / pi * 180
> α
[1] -0.3903895
```

The four parameters for similarity transformation are as follows:

a= 0.3048326, b= -0.002077034, c= -1023524, d= -1760591

Scale Factor, μ = 0.3048255

Rotation Angle, α = -0.3903895 rd

Test of the four parameter values using CP 3:

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} 0.3048326 & -0.002077034 \\ 0.002077034 & 0.3048326 \end{bmatrix} * \begin{bmatrix} 3405960.296 \\ 5757978.003 \end{bmatrix} + \begin{bmatrix} -1023524 \\ -1760591 \end{bmatrix} = \begin{bmatrix} 2764.216 \\ 1702.701 \end{bmatrix}$$

R-Codes:

```
> Z <- matrix(c(-1023524+(0.3048326*3405960.296)+(-0.002077034*5757978.003),
+ -1760591-(-0.002077034*3405960.296)+(0.3048326*5757978.003)))
> Z
          [,1]
[1,] 2764.216
[2,] 1702.701
```

	CP3_X	CP3_Y
Values Acquired from the Image	**2769**	**1721**
Transformed Using Similarity Transformation	**2764**	**1703**

3. General Equations for Affine Transformation:

$$Xn = a \cdot X_0 + b \cdot Y_0 + c$$

$$Yn = d \cdot X_0 + e \cdot Y_0 + f$$

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} X_0 & Y_0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_0 & Y_0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

With six parameters to be determined, a minimum of three 2D control points must be found (Source: Iliffe & Lott, 2000, p-116).

For this affine transformation we have considered Control Points 1, 2 and 3. Control Point (CP) 4 has been used for testing the parameter values through this transformation.

Control Point	X_o (Shape)	Y_o (Shape)	X_n (Image)	Y_n (Image)
1	3405650.201	5758878.930	2668	1461 (3438-1461=**1977**)
2	3405415.509	5758618.090	2597	1541 (3438-1541=**1897**)
3	3405960.296	5757978.003	2769	1721 (3438-1721=**1717**)
4	3405604.353	5758632.242	2654	1533 (3438-1533=**1905**)

R-Codes:

```
> F <- matrix(c(1,3405650.201,5758878.930,0,0,0,
+ 0,0,0,1,3405650.201,5758878.930,
+ 1,3405415.509,5758618.090,0,0,0,
+ 0,0,0,1,3405415.509,5758618.090,
+ 1,3405960.296,5757978.003,0,0,0,
+ 0,0,0,1,3405960.296,5757978.003),
+ nrow=6,ncol=6,byrow=TRUE)
> F
      [,1]      [,2]      [,3] [,4]      [,5]      [,6]
[1,]     1 3405650 5758879    0       0       0
[2,]     0       0       0    1 3405650 5758879
[3,]     1 3405416 5758618    0       0       0
[4,]     0       0       0    1 3405416 5758618
[5,]     1 3405960 5757978    0       0       0
[6,]     0       0       0    1 3405960 5757978
```

```
> G <- c(2668,1977,2597,1897,2769,1717)
> G
[1] 2668 1977 2597 1897 2769 1717
> E = F^-1 * G
> E <- solve(F,G)
> E
[1] -1.016232e+06  3.089386e-01 -5.771466e-03 -1.738425e+06  1.455829e-02
[6]  2.936025e-01

> para_a<-E[1]
> para_a
[1] -1016232
> para_b<-E[2]
> para_b
[1] 0.3089386
> para_c<-E[3]
> para_c
[1] -0.005771466
> para_d<-E[4]
> para_d
[1] -1738425
> para_e<-E[5]
> para_e
[1] 0.01455829
> para_f<-E[6]
> para_f
[1] 0.2936025
```

The six parameters for affine transformation are as follows:

a= -1016232, b= 0.3089386, c= -0.005771466,

d= -1738425, e= 0.01455829, f= 0.2936025

Test of the six parameter values using CP 4:

R-Codes:

```
> Q <- matrix(c(-1016232+(0.3089386*3405604.353)+(-0.005771466*5758632.242),
+ -1738425+(0.01455829*3405604.353)+(0.2936025*5758632.242)))
> Q
          [,1]
[1,] 2654.891
[2,] 1903.599
```

	CP4_X	CP4_Y
Values acquired from the Image	**2654**	**1905**
Transformed Using Affine Transformation	**2655**	**1904**

4. Calculating Similarity and Affine Transformation using CP 4, CP 5

Control Point	X_o (Shape)	Y_o (Shape)	X_n (Image)	Y_n (Image)
1	3405650.201	5758878.930	2668	1461 (3438-1461=**1977**)
2	3405415.509	5758618.090	2597	1541 (3438-1541=**1897**)
3	3405960.296	5757978.003	2769	1721 (3438-1721=**1717**)
4	3405604.353	5758632.242	2654	1533 (3438-1533=**1905**)
5	3405982.652	5758915.909	2766	1444 (3438-1444=**1994**)

R-Codes:

```
> y <- matrix(c(-1023524+(0.3048326*3405604.353)+(-0.002077034*5758632.242),
+ -1760591-(-0.002077034*3405604.353)+(0.3048326*5758632.242),
+ -1023524+(0.3048326*3405982.652)+(-0.002077034*5758915.909),
+ -1760591-(-0.002077034*3405982.652)+(0.3048326*5758915.909),
+ -1016232+(0.3089386*3405604.353)+(-0.005771466*5758632.242),
+ -1738425+(0.01455829*3405604.353)+(0.2936025*5758632.242),
+ -1016232+(0.3089386*3405982.652)+(-0.005771466*5758915.909),
+ -1738425+(0.01455829*3405982.652)+(0.2936025*5758915.909)))
> y
          [,1]
[1,] 2654.355
[2,] 1901.395
[3,] 2769.083
[4,] 1988.651
[5,] 2654.891
[6,] 1903.599
[7,] 2770.125
[8,] 1992.391
```

Table: Comparison of Similarity and Affine Transformation

Comparison	CP4_X	CP4_Y	CP5_X	CP5_Y
Values Acquired from the Image	2654	1905	2766	1994
Transformed Using Similarity Transformation	2654	1901	2769	1989
Transformed using Affine Transformation	2655	1904	2770	1992
Difference_ Similarity Transformation	0 (2654-2654=0)	4 (1905-1901=4)	-3	5
Difference_ Affine Transformation	-1 (2654-2655= -1)	1 (1905-1904=1)	-4	2

From the above table, it is clear that the differences between the original and transformed coordinates are negligible in both cases (similarity and affine transformation).

In general observation, it can be summarized that the differences are smaller in case of affine transformation. Affine transformation is always advantageous transforming coordinates between an image and a map. The reasons can be as follows:

a) It corrects affinity due to non-orthogonality
b) It allows two different scale factors
c) It enables scale and rotation adjustments in each direction
d) It allows shearing which means the angle between the coordinate axes can be changed.
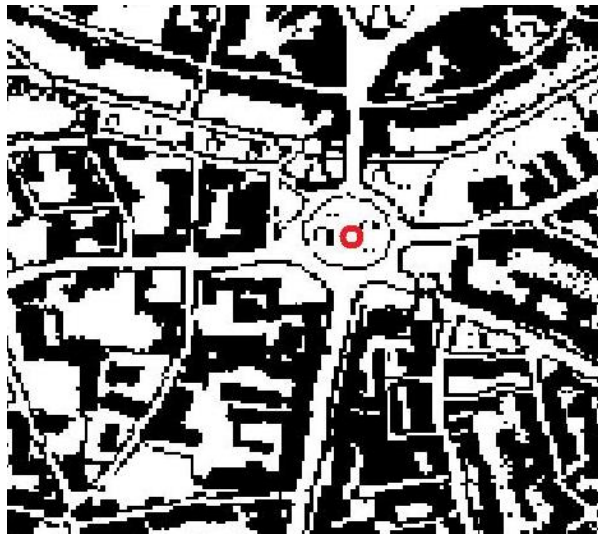
CP 1

CP 2

CP 3

CP 4

CP 5

Coordinate Transformations II

Course *Reference Systems for Geoinformation*

Lab 4

Goals

To understand how to optimize transformations when using a larger number of common points than is necessary to solve for the transformation parameters. As a side effect, this lab will introduce you to the method of parameter estimation through least squares adjustment, which is widely used for solving over-determined problems.

Tasks (15 points)

Same as 2nd part of lab3, but now using a minimum of *three* points for the similarity transformation. This leads to a "degree of freedom" of at least two, i.e. you have two equations more than you need to solve for the four parameters. This overdetermination allows for computing not only the parameters, but also their accuracy and other statistical properties.

1. Describe how the least squares solution takes you from an overdetermined to an exactly determined problem. (5 points)
2. Before you start any calculations, write down a precise procedure of how to get from your measurements to the solution for the parameters (5 points); decide on the tool to use.
3. Test the parameters you calculated with at least two sample points (different from the ones you used to determine the parameters!). Comment on the results! (2 points)
4. Submit all data and results in the form of a technical report, documenting your complete procedure and results (3 points for the form and completeness).

Use Iliffe Appendix E.2.1 for the procedure and formulae.

Schedule

Handout: May 31st
Lab sessions: May 31st and June 7th
Submission: until June 10th 12:00 o'clock (noon)

Reference Systems for Geographic Information

"Coordinate Transformations II"

Technical Report on Lab 4

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1. *Describe how the least squares solution takes you from an overdetermined to an exactly determined problem. (5 points)*

Answer: Let's start with some theoretical background (extracted from various online sources):

Overdetermined System of Equations

A linear system of equations in which there are more equations than there are variables. For example, a system with three equations and only two unknowns is overdetermined.

Consider the system of 3 equations and 2 unknowns (x_1 and x_2):

$2x_1 + x_2 = -1$

$-3x_1 + x_2 = -2$

$-x_1 + x_2 = 1$

Least Squares Method

The least squares parameter estimation method is a variation of the probability plotting methodology in which one mathematically fits a straight/curve line to a set of points in an attempt to estimate the parameters. The method of least squares requires that a straight/curve line be fitted to a set of data points such that the sum of the squares of the vertical deviations from the points to the line/curve is minimized, if the regression is on Y, or the line/curve be fitted to a set of data points such that the sum of the squares of the horizontal deviations from the points to the line is minimized, if the regression is on X.
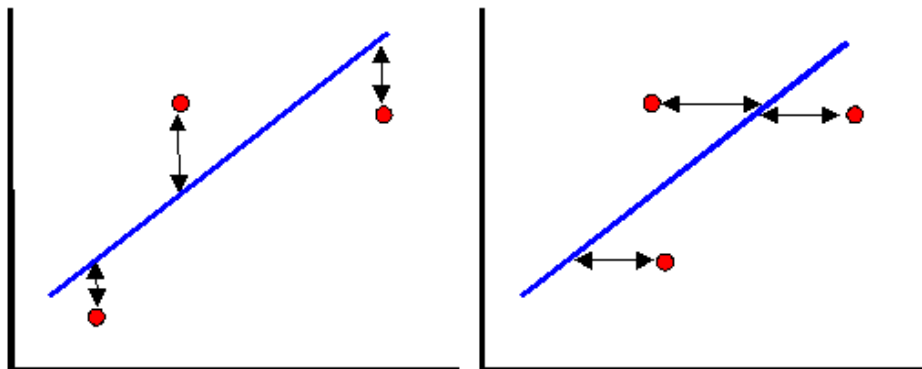

Figure 01: Linear regression on Y and on X

How the least squares solution takes from an overdetermined to an exactly determined problem?

The linear least squares fitting technique is the simplest and most commonly applied form of linear regression and provides a solution to the problem of finding the best fitting straight line through a set of points.

Therefore it is better to have many points or values (overdetermined) to achieve the best fit line or get the minimum errors. In this way, it is possible to gain better accuracy level using many points and ultimately reaching to an exactly determined system

Again, least square is a method that is much useful to determine a line or curve that best represents many measured values. This is then useful for best estimation of unobserved parameters. This situation is described below in this lab's context:

'x' coordinate of the shapefile/source system
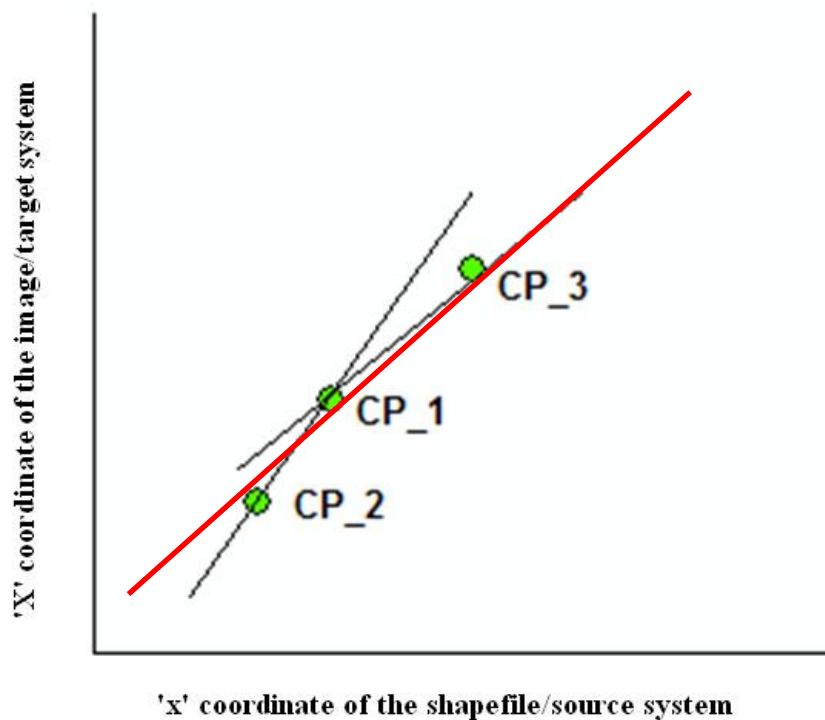
I measured values of the control points (CPs) coordinates both from the image (target system) and the shapefile (source system). The above figure is drawn considering only the X-axis values from the both coordinate systems (image and shapefile). At first, a line is fitted through CP_1 and CP_2. Then a second line is fitted through CP_1 and CP_3. Finally, the best fitted line (the red line) is drawn with the least possible square distances from all the points. The function of this red line or the best fitted line allows determining the best value for unknown points within the same area.

The more control points will be used the better would be the accuracy or result. If I consider infinite number of CPs then the line/curve would be exactly determined. The thing is that when we use more than the required numbers of known points to calculate unknown parameters, it is called overdetermined problem.

Therefore, from the above discussion, it is clear that using overdetermined problem/system leads us towards an exactly determined problem/system. Because using an overdetermined system allows more points along the best fitted line/curve and finally the ultimate result is we are getting exactly determined system.

2. Before you start any calculations write down a precise procedure of how to get from your measurements to the solution for the parameters (5 points); decide on the tool to use.

Answer:

The steps to get the solution for the parameters are as follows:

i. At first, Munster image is opened in MS Paint to get the image coordinates.

ii. Then the ms_strassen shapefile is opened in ArcMap to get the shape coordinates.

iii. The coordinates of 5 different control points have been noted down from both the image and the shapefile [the exact same position of the control points has been tried to get for both the image and the shapefile].

iv. The image coordinates represent the monitor coordinate while the shapefile coordinates represent the real world coordinate values.

v. The first 3 control points have been used for calculating the parameters. The last 2 control points have been used for testing the parameter values.

vi. The calculations are performed in R-software.

vii. The ms_strassen shapefile coordinates are used to create the matrix A, with two more columns with 1, 0 and 0, 1 sequence altering in each row.

viii. R cannot handle huge numbers and moreover the shapefile coordinates are much bigger than the monitor coordinates. Therefore the coordinate values measured from the ms_strassen shapefile have to be reduced.

ix. This has been done by subtracting a certain value (3400500 for the x-values and 5755000 for the y-values) from each coordinate value of the ms_strassen shapefile.

x. To do this, a matrix B has been developed with two extra column with all zeros.

xi. Then a new shrink matrix 'C' has been created (by subtracting matrix B from matrix A). The new values of C are now comparable with the image coordinates.

xii. Another matrix 'D' has been created by using the image coordinates for the first three points.

xiii. Finally, the values of the parameters (a, b, Δ x, Δ y) have been calculated using R (the basic equations are given as follows).

General Equations for Similarity Transformation:

$$Xn = a \cdot X_0 + b \cdot Y_0 + c$$
$$Yn = -b \cdot X_0 + a \cdot Y_0 + d$$

$$\begin{bmatrix} Xn \\ Yn \end{bmatrix} = \begin{bmatrix} X_0 & Y_0 & 1 & 0 \\ Y_0 & -X_0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$A := \begin{pmatrix} x_1 & y_1 & 1 & 0 \\ y_1 & -x_1 & 0 & 1 \\ x_2 & y_2 & 1 & 0 \\ y_2 & -x_2 & 0 & 1 \\ x_3 & y_3 & 1 & 0 \\ y_3 & -x_3 & 0 & 1 \end{pmatrix}$$

$$D := \begin{pmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \end{pmatrix}$$

Source System   Target System

$$s = (A^T \cdot A)^{-1} \cdot A^T \cdot D$$

$$s = \begin{pmatrix} a \\ b \\ \Delta x \\ \Delta y \end{pmatrix}$$

Table 01: The first three control point's coordinates

Control Point	x (Shape)	y (Shape)	X (Image)	Y (Image)
1	3405650.201	5758878.930	2668	1461 (3438-1461=**1977**)
2	3405415.509	5758618.090	2597	1541 (3438-1541=**1897**)
3	3405960.296	5757978.003	2769	1721 (3438-1721=**1717**)

There is a shift of the origin between the two coordinate systems and therefore I have subtracted the y-value from the maximum y-value for the image coordinate system. The reason for this subtraction is that the origin of the image is in the upper left corner and the origin of the shapefile coordinates is in the lower left corner. This is why I have to make an adjustment in vertical direction or the y-values.

R-code:

> A <-matrix (c (3405650.201, 5758878.930, 3405415.509, 5758618.090, 3405960.296, 5757978.003, 5758878.930, -3405650.201, 5758618.090, -3405415.509, 5757978.003, -3405960.296, 1, 0, 1, 0, 1,0, 0, 1, 0, 1, 0, 1), 6)

```
> A
        [,1]      [,2] [,3] [,4]
[1,] 3405650  5758879    1    0
[2,] 5758879 -3405650    0    1
[3,] 3405416  5758618    1    0
[4,] 5758618 -3405416    0    1
[5,] 3405960  5757978    1    0
[6,] 5757978 -3405960    0    1
```

> B <-matrix (c (3400500, 5755000, 3400500, 5755000, 3400500, 5755000, 5755000, -3400500, 5755000, -3400500, 5755000, -3400500, 0, 0, 0, 0, 0,0, 0, 0, 0, 0, 0, 0), 6)

```
> B
        [,1]      [,2] [,3] [,4]
[1,] 3400500  5755000    0    0
[2,] 5755000 -3400500    0    0
[3,] 3400500  5755000    0    0
[4,] 5755000 -3400500    0    0
[5,] 3400500  5755000    0    0
[6,] 5755000 -3400500    0    0
```

```
> C<-A-B
> C
         [,1]       [,2] [,3] [,4]
[1,] 5150.201   3878.930    1    0
[2,] 3878.930  -5150.201    0    1
[3,] 4915.509   3618.090    1    0
[4,] 3618.090  -4915.509    0    1
[5,] 5460.296   2978.003    1    0
[6,] 2978.003  -5460.296    0    1
```

> D<-c (2668, 1977, 2597, 1897, 2769, 1717)

> D

[1] 2668 1977 2597 1897 2769 1717

> S <- solve (t(C) %*% C) %*% (t(C) %*% D)

> S <- solve (t(C) %*% C) %*% (t(C) %*% D)

> S

     [,1]

[1,]   0.29469040

[2,] -0.01304092

[3,] 1198.41299630

[4,] 767.21262585

Therefore, **a= 0.29469040, b= -0.01304092,**

Δ x= 1198.41299630 and Δ y= 767.21262585.

3. *Test the parameters you calculated with at least two sample points (different from the ones you used to determine the parameters!). Comment on the results! (2 points)*

Answer:

Control Point	x (Shape)	y (Shape)	X (Image)	Y (Image)
4	3405604.353	5758632.242	2654	1533 (3438-1533=**1905**)
5	3405982.652	5758915.909	2766	1444 (3438-1444=**1994**)

Using the above two control points (CP_4 and CP_05) I have tested the parameters values.

R-code:

> X4 <- 1198.41299630+ 0.29469040*(3405604.353-3400500) - 0.01304092*(5758632.242-5755000)

> X4

[1] 2655.249

> Y4 <- 767.21262585+0.01304092*(3405604.353-3400500) +
0.29469040*(5758632.242-5755000)

> Y4

[1] 1904.165

> X5 <- 1198.41299630+ 0.29469040*(3405982.652-3400500) -
0.01304092*(5758915.909-5755000)

> X5

[1] 2763.031

> Y5 <- 767.21262585+0.01304092*(3405982.652-3400500) +
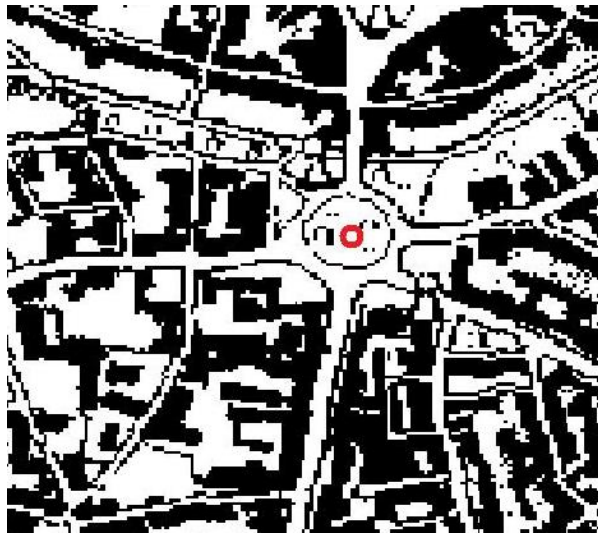0.29469040*(5758915.909-5755000)

> Y5

[1] 1992.692

	CP4_X	CP4_Y	CP5_X	CP5_Y
Values acquired from the Image	2654	1905	2766	1994
Similarity Transformation (parameter estimation through least squares adjustment)	2655	1904	2763	1993

Comment

Most of the values are same. A slight variation is also found. This is may be due to taking the positions of the same point in the image and shapefile. More or less the result is acceptable.

[The positions of the control points are shown in the image below].

CP 1

CP 2

CP 3

CP 4

CP 5